NAG Toolbox for MATLAB

f07gd

1 Purpose

f07gd computes the Cholesky factorization of a real symmetric positive-definite matrix, using packed storage.

2 Syntax

```
[ap, info] = f07gd(uplo, n, ap)
```

3 Description

f07gd forms the Cholesky factorization of a real symmetric positive-definite matrix A either as $A = U^{\mathrm{T}}U$ if **uplo** $=$ 'U' or $A = LL^{\mathrm{T}}$ if **uplo** $=$ 'L', where U is an upper triangular matrix and L is lower triangular, using packed storage.

4 References

Demmel J W 1989 On floating-point errors in Cholesky *LAPACK Working Note No. 14* University of Tennessee, Knoxville

Golub G H and Van Loan C F 1996 *Matrix Computations* (3rd Edition) Johns Hopkins University Press, Baltimore

5 Parameters

5.1 Compulsory Input Parameters

1: **uplo – string**

Indicates whether the upper or lower triangular part of A is stored and how A is to be factorized.

uplo $=$ 'U'

The upper triangular part of A is stored and A is factorized as $U^{\mathrm{T}}U$, where U is upper triangular.

uplo $=$ 'L'

The lower triangular part of A is stored and A is factorized as LL^{T}, where L is lower triangular.

Constraint: **uplo** $=$ 'U' or 'L'.

2: **n – int32 scalar**

n, the order of the matrix A.

Constraint: $\mathbf{n} \geq 0$.

3: **ap**$(*)$ **– double array**

Note: the dimension of the array **ap** must be at least $\max(1, \mathbf{n} \times (\mathbf{n}+1)/2)$.

The n by n symmetric matrix A, packed by columns.

More precisely,

if **uplo** $=$ 'U', the upper triangle of A must be stored with element A_{ij} in $\mathbf{ap}(i + j(j-1)/2)$ for $i \leq j$;
if **uplo** $=$ 'L', the lower triangle of A must be stored with element A_{ij} in $\mathbf{ap}(i + (2n - j)(j-1)/2)$ for $i \geq j$.

5.2 Optional Input Parameters

None.

5.3 Input Parameters Omitted from the MATLAB Interface

None.

5.4 Output Parameters

1: **ap**($*$) – **double array**

Note: the dimension of the array **ap** must be at least $\max(1, \mathbf{n} \times (\mathbf{n}+1)/2)$.

If **info** $= 0$, the factor U or L from the Cholesky factorization $A = U^{\mathrm{T}}U$ or $A = LL^{\mathrm{H}}$, in the same storage format as A.

2: **info** – **int32 scalar**

info $= 0$ unless the function detects an error (see Section 6).

6 Error Indicators and Warnings

Errors or warnings detected by the function:

info $= -i$

If **info** $= -i$, parameter i had an illegal value on entry. The parameters are numbered as follows:

1: **uplo**, 2: **n**, 3: **ap**, 4: **info**.

info > 0

If **info** $= i$, the leading minor of order i is not positive-definite and the factorization could not be completed. Hence A itself is not positive-definite. This may indicate an error in forming the matrix A. To factorize a symmetric matrix which is not positive-definite, call f07pd instead.

7 Accuracy

If **uplo** $=$ 'U', the computed factor U is the exact factor of a perturbed matrix $A + E$, where

$$|E| \leq c(n)\epsilon\left|U^{\mathrm{T}}\right||U|,$$

$c(n)$ is a modest linear function of n, and ϵ is the ***machine precision***.

If **uplo** $=$ 'L', a similar statement holds for the computed factor L. It follows that $\left|e_{ij}\right| \leq c(n)\epsilon\sqrt{a_{ii}a_{jj}}$.

8 Further Comments

The total number of floating-point operations is approximately $\frac{1}{3}n^3$.

A call to f07gd may be followed by calls to the functions:

f07ge to solve $AX = B$;

f07gg to estimate the condition number of A;

f07gj to compute the inverse of A.

The complex analogue of this function is f07gr.

9 Example

```
uplo = 'L';
n = int32(4);
ap = [4.16;
     -3.12;
     0.5600000000000001;
     -0.1;
     5.03;
     -0.83;
     1.18;
     0.76;
     0.34;
     1.18];
[apOut, info] = f07gd(uplo, n, ap)
```

```
apOut =
    2.0396
   -1.5297
    0.2746
   -0.0490
    1.6401
   -0.2500
    0.6737
    0.7887
    0.6617
    0.5347
info =
           0
```